

4th Scientific Colloquium

**Simplified model for a rivulet spreading
down an inclined wetted plate**



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Outline

① Introduction

- Why to study rivulet interface – Coordinate system
- Basic principle – Assumptions

② Static rivulet

③ Spreading rivulet

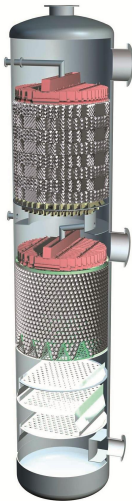
④ Velocity field

⑤ Conclusions

⑥ Discussion

Why to study rivulets

Numerous applications in mass transfer and reaction engineering



[Sulzer ChemTech]

Hydrodynamics

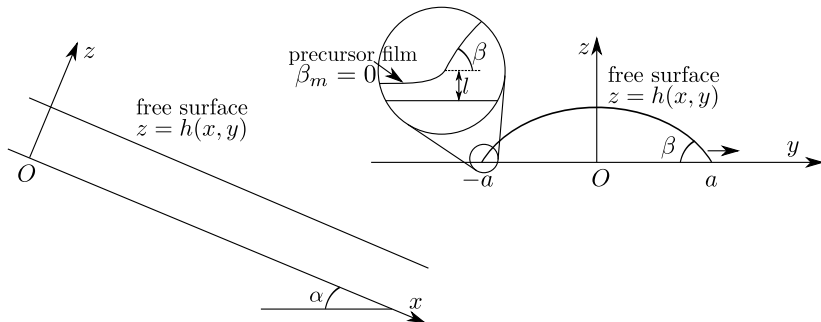
- Fuel cells
 - water management inside PEMFC fuel cells
- Aerospace engineering
 - in flight formation of rivulets on plane wings

Gas-liquid interface

- Packed columns
 - wetting performance
 - mass transfer coefficients
- Catalytic reactors
 - wetting of the catalyst

Used coordinate system

Cartesian coordinate system and basic notations



Notations

$a[\text{m}]$ half-width of the rivulet

$h[\text{m}]$ height

$l[\text{m}]$.. intermediate region length
scale

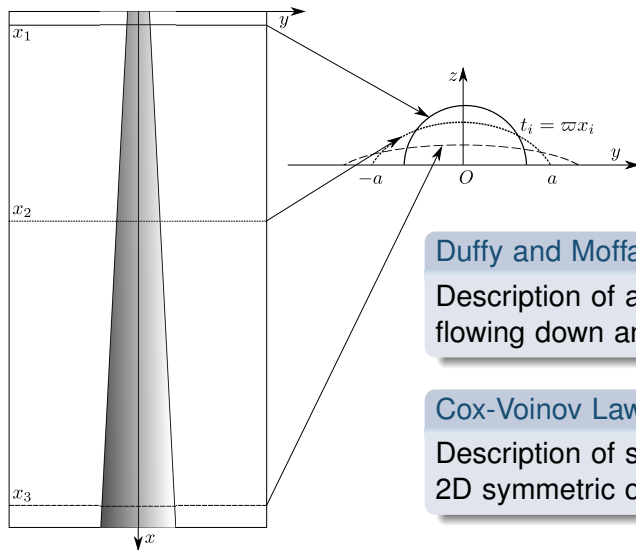
$x, y, z[\text{m}]$ coordinate system

$\alpha[-]$ plate inclination angle

$\beta[-]$ dynamic contact angle

Proposed method principle

Parallel between spreading of trickle in time and along the plate



Duffy and Moffat[11]

Description of an uniform rivulet flowing down an inclined plate

Cox-Voinov Law[24]

Description of spreading of the 2D symmetric object in time

Simplifying assumptions

Reduce problem to one spatial and one time coordinate

- Newtonian liquid, ρ , μ and γ are constant
- $h_t(t, x, y) = 0$, Q is constant
- $\mathbf{u} = (u, v, w)$, $u \gg v \sim w$
- $z = h : u_x = v_y = 0$
- Gravity is the only acting body force.
- The rivulet is shallow. Its dynamic contact angles are assumed small, $\beta(x) \ll 1$, and its GL interface is nearly flat, $h_y(x, y) \ll 1$.
- There is a thin precursor film of height l on the whole studied surface. Thus there is no contact angle hysteresis and $\beta_m = 0$. The height of the precursor film, l , can also be taken as the intermediate region length scale well separating the inner and outer solution for the profile shape[24].

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Outline

- ① Introduction
- ② Static rivulet
 - Gas-liquid interface
 - Liquid flow rate
- ③ Spreading rivulet
- ④ Velocity field
- ⑤ Conclusions
- ⑥ Discussion

Simplified Navier-Stokes equations

Assumption of very shallow and nearly flat rivulet [11, 12]

Simplified NS

$$0 = -p_x + \rho g \sin \alpha + \mu u_{zz}$$

$$0 = -p_y$$

$$0 = -p_z - \rho g \cos \alpha$$

Used boundary conditions

$$z = 0 : u = u(y, z) = 0$$

$$z = h : p = p_A - \gamma h_{yy} \quad \text{and} \quad u_z = 0$$

$$y = \pm a : h = 0 \quad \text{and} \quad h_y = \mp \tan \beta$$

Notes

$$h = h(y), \quad h_y = \frac{dh}{dy}, \quad p = p(y, z)$$

Legend to results of integration

In the presented case, there exist a physical solution for $\alpha \in (0, \pi)$

Notation

The problem decomposes to three cases which will be denoted:

- (i) $\iff \alpha \in (0, \pi/2)$
- (ii) $\iff \alpha = \pi/2$
- (iii) $\iff \alpha \in (\pi/2, \pi)$

Dimensionless numbers

$$B = a^2 \frac{\rho g \cos |\alpha|}{\gamma}$$

$$\zeta = \frac{y}{a}$$

$\rho[\text{kg m}^{-3}]$ liquid density

$\gamma[\text{N m}^{-1}]$ surface tension

$g[\text{m s}^{-2}]$ gravitational acceleration

Integration results

Rectilinear rivulet gas-liquid interface shape

Gas-liquid interface shape for the three cases

$$h(\zeta) = \begin{cases} \frac{a \tan \beta}{\sqrt{B}} \left(\frac{\cosh \sqrt{B} - \cosh \sqrt{B}\zeta}{\sinh \sqrt{B}} \right) & (i) \\ \frac{a \tan \beta}{2} (1 - \zeta^2) & (ii) \\ \frac{a \tan \beta}{\sqrt{B}} \left(\frac{\cos \sqrt{B}\zeta - \cos \sqrt{B}}{\sin \sqrt{B}} \right) & (iii) \end{cases}$$

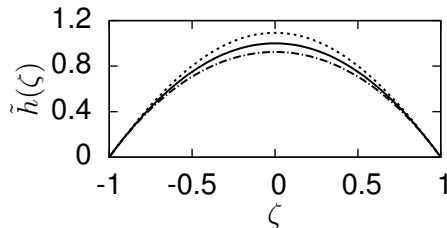
Scaling of the GL interface height

$$\tilde{h}(\zeta) = \frac{2h(\zeta)}{a \tan \beta}$$

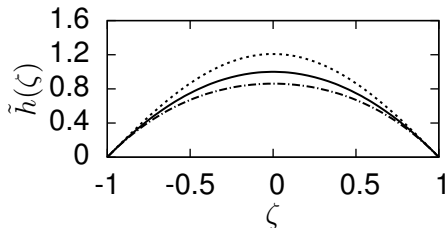
Effects of Bond number on GL interface shape

Dependence of GL interface shape on the ratio of surface and volumetric forces

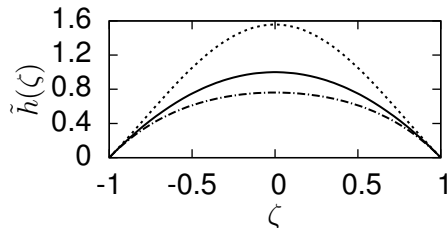
$B = 0.2$



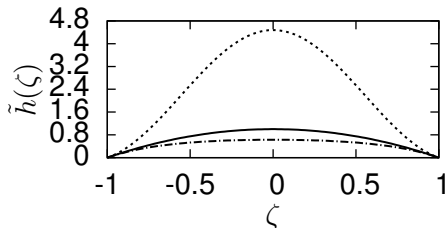
$B = 2.0$



$B = 4.0$



$B = 8.0$



Liquid volumetric flow rate

Integration of velocity field on a domain of one transversal cut

Velocity field integration

$$\frac{Q}{a} = \int_{-1}^1 \int_0^{h(\zeta)} u(\zeta, z) dz d\zeta = \int_{-1}^1 \int_0^{h(\zeta)} \frac{\rho g \sin \alpha}{2\mu} (2h(\zeta)z - z^2) dz d\zeta$$

Dimensionless liquid flow rate

$$\frac{\mu Q}{a^4 \rho g \sin \alpha \tan^3 \beta} = F(B)$$

$$F(B) = \begin{cases} \frac{54\sqrt{B} \cosh \sqrt{B} + 6\sqrt{B} \cosh 3\sqrt{B} - 27 \sinh \sqrt{B} - 11 \sinh 3\sqrt{B}}{36B^2 \sinh^3 \sqrt{B}} & (i) \\ \frac{4}{105} & (ii) \\ \frac{27 \sin \sqrt{B} + 11 \sin 3\sqrt{B} - 54\sqrt{B} \cos \sqrt{B} - 6\sqrt{B} \cos 3\sqrt{B}}{36B^2 \sin^3 \sqrt{B}} & (iii) \end{cases}$$

Liquid flow rate scaling and properties

Maximal flow rate occurs at $\alpha = \pi/2$

Scaling

$$\tilde{Q} = \frac{105\mu}{4a^4\rho g \sin \alpha \tan^3 \beta} Q = \frac{105\rho g \mu \cos^2 \alpha}{4\gamma^2 \sin \alpha \tan^3 \beta} \frac{Q}{B^2}$$

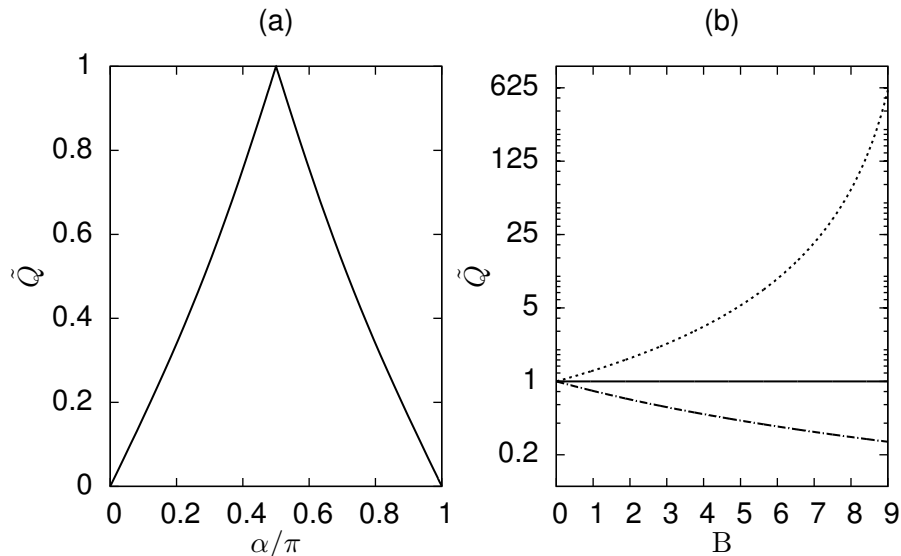
Asymptotic behavior

$$(i) : \quad \lim_{B \rightarrow \infty} \tilde{Q}(B) = 0$$

$$(iii) : \quad \lim_{B \rightarrow \pi_-^2} \tilde{Q}(B) = \infty$$

Liquid flow rate scaling and properties

Maximal flow rate occurs at $\alpha = \pi/2$



Outline

- ① Introduction
- ② Static rivulet
- ③ Spreading rivulet
 - Problem definition
 - Basics
 - Gas-liquid interface
- ④ Velocity field
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- ⑥ Discussion

Difference from static rivulet

Changed boundary conditions

Simplified NS

$$0 = -p_x + \rho g \sin \alpha + \mu u_{zz}$$

$$0 = -p_y$$

$$0 = -p_z - \rho g \cos \alpha$$

Used boundary conditions

$$z = 0 : \quad \mathbf{u}(x, y, z) + \lambda \nabla \mathbf{u} = \mathbf{0}, \quad \lambda \ll 1$$

$$z = h : \quad p = p_A - \gamma \kappa \quad \text{and} \quad u_z = 0$$

$$y = \pm a(x) : \quad h = 0 \quad \text{and} \quad h_y = \mp \tan \beta(x)$$

Notes

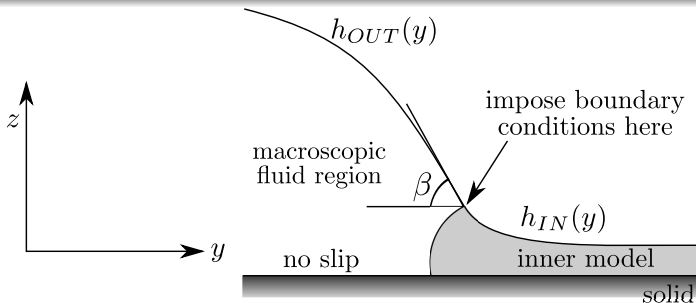
$$h = h(x, y), \quad h_y = \frac{\partial h}{\partial y}, \quad p = p(x, y, z)$$

Cox-Voinov law

Solution of thin film governing equation for spreading of symmetric object[24]

Thin film governing equation - outer and inner

$$h_t + \frac{\gamma}{3\mu} \frac{\partial}{\partial a} (h^3 h_{aaa}) = 0, \quad h_t + \frac{1}{3\mu} \frac{\partial}{\partial a} [(h^3 + 3\lambda h^2) \gamma h_{aaa}] = 0$$

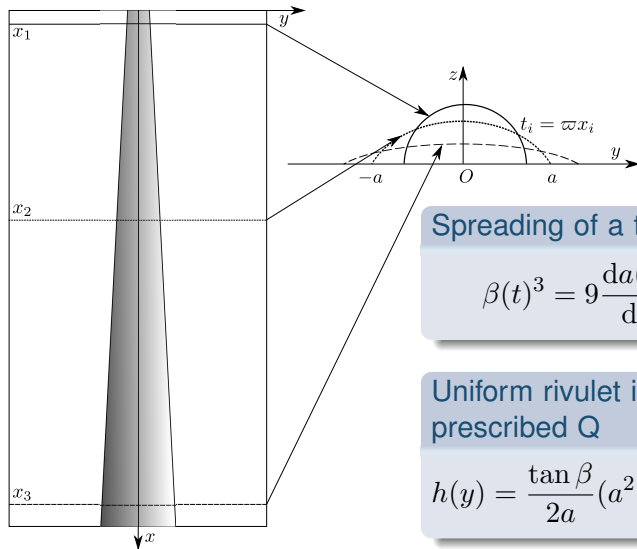


Cox-Voinov law[24]

$$\beta(t)^3 = 9 \frac{da(t)}{dt} \frac{\mu}{\gamma} \ln \left(\frac{a(t)}{2e^2 l} \right)$$

Basic principle remainder

Combination of Cox-Voinov law with unidirectional rivulet flow description



Spreading of a trickle in time

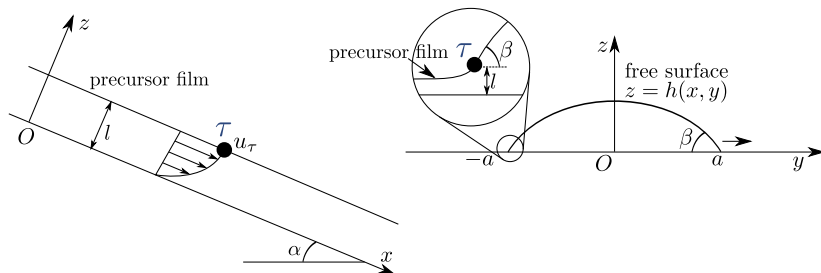
$$\beta(t)^3 = 9 \frac{da(t)}{dt} \frac{\mu}{\gamma} \ln \left(\frac{a(t)}{2e^2 l} \right)$$

Uniform rivulet interface with prescribed Q

$$h(y) = \frac{\tan \beta}{2a} (a^2 - y^2), \quad a \approx \eta \frac{1}{\beta^{3/4}}$$

Transformation from t to x

Presence of falling thin liquid film, l , on all the plate



From t to x using u_τ

$$u_\tau = \frac{\rho g \sin \alpha}{2\mu} l^2 \implies t = \frac{2\mu}{\rho g l^2 \sin \alpha} x = \varpi x$$

Modeling method for the case of no gravity effects

Gas-liquid interface shape defined implicitly by 1 NAE

- $\beta \ll 1 \implies a \doteq \eta \frac{1}{\beta^{3/4}}$

-

$$\beta(t)^3 = 9 \frac{da(t)}{dt} \frac{\mu}{\gamma} \ln \left(\frac{a(t)}{2e^2 l} \right) \rightsquigarrow \beta^{19/4} = -A \frac{d\beta}{dt} \ln \left(\frac{B}{\beta^{3/4}} \right)$$

$$\beta = \beta(t), \quad A = \frac{27}{4} \frac{\eta \mu}{\gamma}, \quad B = \frac{\eta}{2e^2 l}, \quad \eta = \left(\frac{4\mu Q}{105\rho g \sin \alpha} \right)^{\frac{1}{4}}$$

-

$$\beta(0) = \beta_0, \quad t = \varpi x \rightsquigarrow x - \frac{\bar{A}}{\beta_0^{15/4}} \left[\ln \left(\frac{B}{\beta_0^{3/4}} \right) - \frac{1}{5} \right] + \bar{C} = 0$$

$$\bar{A} = \frac{4}{15} \frac{A}{\varpi}, \quad \bar{C} = \frac{4A}{15\varpi\beta_0^{15/4}} \left[\ln \left(\frac{B}{\beta_0^{3/4}} \right) - \frac{1}{5} \right]$$

Modeling method gravity effects

1 ODE with 1 NAE nested in each time step has to be solved numerically

- $t = \varpi x, \quad \beta(0) = \beta_0$

-

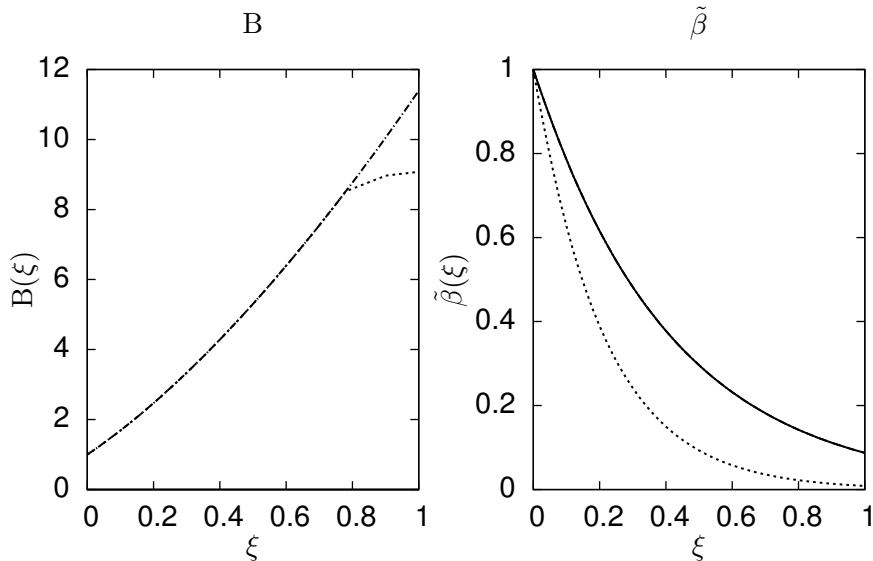
$$\frac{\mu Q}{a^4 \rho g \sin \alpha \tan^3 \beta} = F(B)$$

-

$$\beta(\varpi x)^3 = \frac{9}{\varpi} \frac{da(\varpi x)}{dx} \frac{\mu}{\gamma} \ln \left(\frac{a(\varpi x)}{2e^2 l} \right)$$

B and $\tilde{\beta}$ along the plate

$\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1$ m, $\beta_0 = 0.05$, $Q = 0.01$ ml s⁻¹, $\alpha/\pi = \{1/3, 2\pi/3\}$



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- ① Introduction
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- ④ Velocity field
 - In x axis direction
 - In y and z directions
- ⑤ Conclusions
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Velocity in x -axis direction

Approximated through the static rivulet description

Dimensional $u(\mathbf{x})$

$$\mathbf{u}(\mathbf{x}) = (u(\mathbf{x}), v(\mathbf{x}), w(\mathbf{x}))$$

$$u(\mathbf{x}) = \frac{\rho g \sin \alpha}{2\mu} (2h(x, y)z - z^2)$$

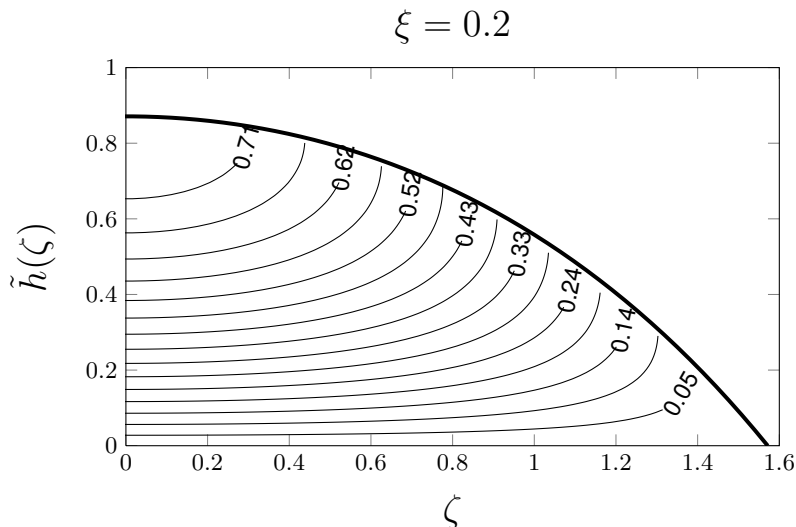
Scaling

$$\xi = \frac{x}{L}, \quad \zeta = \frac{y}{a(0)}, \quad \tilde{z} = \frac{z}{h(0, 0)}$$

$$\tilde{u} = \frac{u}{u(\mathbf{0})}$$

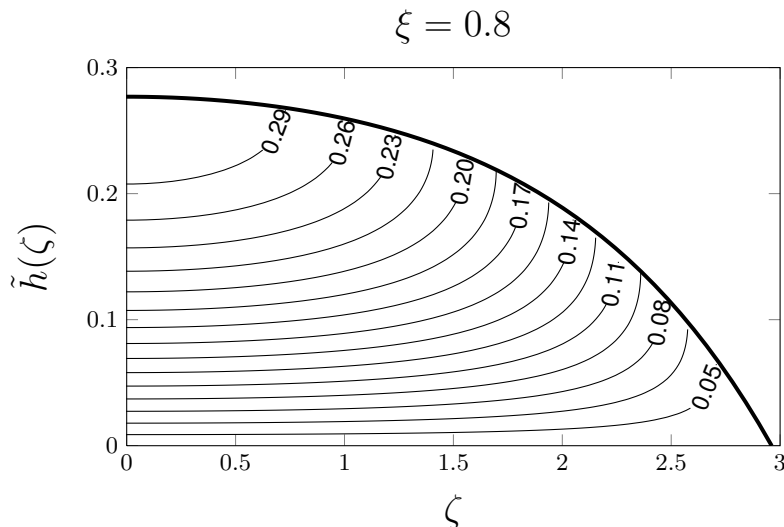
Contours plot of velocity in x -axis direction

Water, $\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1$ m, $\beta_0 = 0.05$, $Q = 0.01$ ml s⁻¹, $\alpha = \pi/3$



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Simulation of velocity field in transversal cut

Numerical PIV based technique – connect the dots

For each subrivulet:

- Get the current width of the rivulet, ζ_i , from the Cox-Voinov law.
- Calculate the current maximal rivulet height, \tilde{h}_i^0 , from description of an uniform rivulet.
- Create a mesh Ω_i^h on a domain Ω_i ,

$$\Omega_i = \left\{ (\zeta, \tilde{z}) : \zeta \in \langle 0; \zeta_i \rangle \mid \tilde{z} \leq \tilde{h}_i(\zeta) \right\}$$

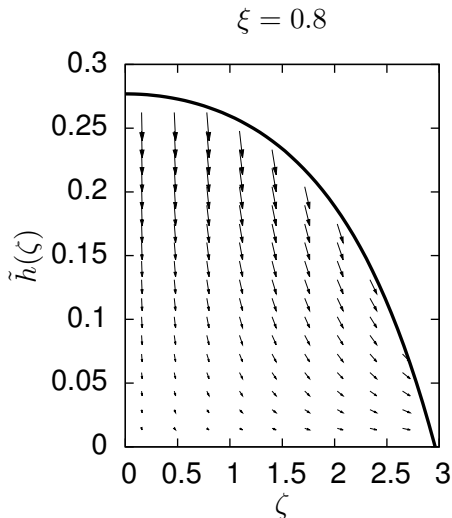
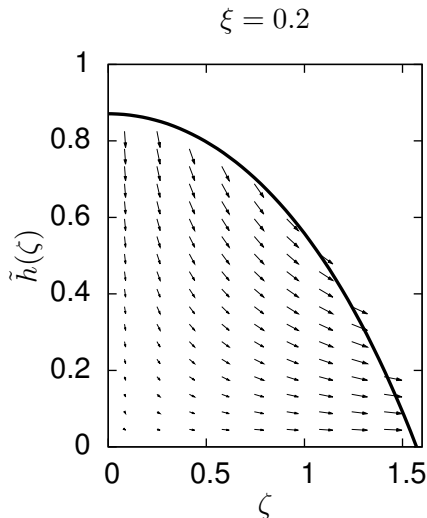
$$\Omega_i^h = \left\{ (\zeta^j, \tilde{z}^k) : \zeta^j \in \langle 0; \zeta_i \rangle \mid \tilde{z}^k \leq \tilde{h}_i(\zeta^j) \right\}_{j=1, \dots, M_1; k=1, \dots, M_2}$$

- Save the current mesh, Ω_i^h .

From the saved meshes, Ω_i^h , $i = 1, \dots, N$, evaluate the velocity field in the $\zeta - \tilde{h}$ plane (connect the dots).

Simulated velocity field v and w components

Water, $\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1$ m, $\beta_0 = 0.05$, $Q = 0.01$ ml s⁻¹, $\alpha = \pi/3$



Conclusions

Simple yet accurate enough method to determine size of $(g) - (l)$ interface of rivulet

Practical advantages of proposed algorithm

- Suitable for parallelization
- Usable for qualitative study of dependence of S_{g-l} on process parameters

Accuracy

- Deviation $< 5\%$ in comparison with experimental data.

Outlook

- Study of spreading on dry plate
- Study of transition states between dry and wetted plate

References I

In order of appearance

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