Gravity-driven rivulet flow on a slowly varying substrate

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Abstract

Rivulet type flow is of great importance in many engineering areas including the packed columns design or heat exchangers calculations. Our concentration lies on the case of a rivulet flowing in the azimuthal direction from top to bottom of a large horizontal cylinder. Simplified solutions to the Navier-Stokes equation for the cases of a rivulet with (i) constant contact angle and varying width, (ii) constant width and varying contact angle, and (iii) varying both contact angle and width were compared to a direct numerical simulation. Such comparison may shed light on the usability of simplified models for the real-life problems.

1. Introduction

Even with the ever-growing power of computers, a seemingly simple problem of a gravity driven spreading flow of a liquid is still too complex for parametric studies via CFD algorithms. Hence, there is still a need for simplified solutions to such a problem.

In our previous work, we derived a computationally inexpensive method to determine the size of the gas-liquid interface of a rivulet flowing down an inclined wetted plate (see [1] and references therein). Also, we concentrated on possibilities of modeling such a flow in OpenFOAM, the most widely used opensource CFD software[2].

In the present work, we generalize the developed method for the case of the flow in the azimuthal direction from the top to the bottom of a large horizontal cylinder. We compare the obtained results with CFD experiment carried out in the OpenFOAM software as well as with results obtained from other simplified analytical solutions to the studied problem[3, 4].

Such a comparison permits to evaluate a legitimacy of assumptions made during the method derivation. Furthermore, the presented results offer, within accuracy limitations of the used CFD methods, a baseline for usability assessment of the simplified models.

2. Coordinate system and simplifying assumptions

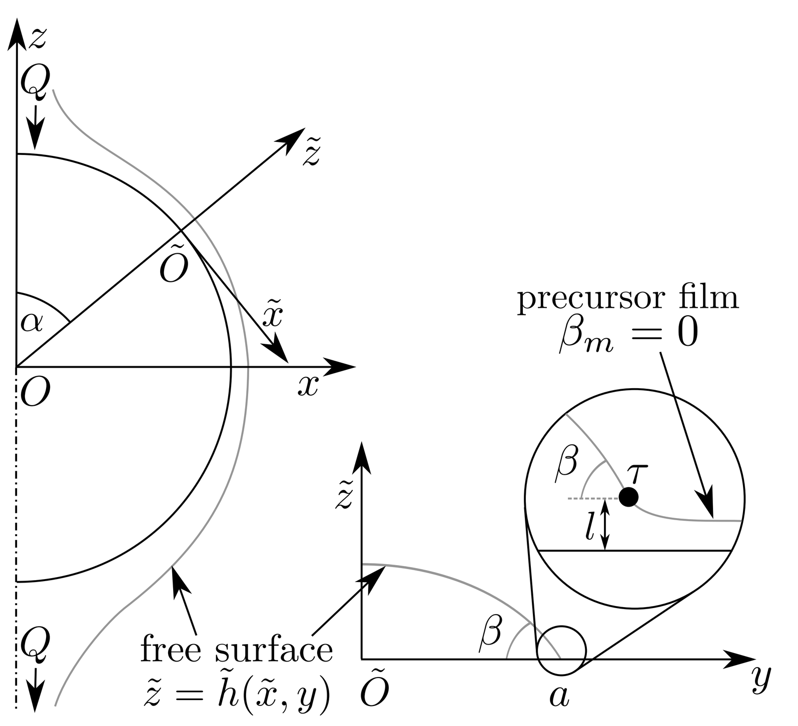

Figure 1: *A particular coordinate system with basics of the used rivulet spreading notation. The global coordinate system is denoted by (O, x, y, z). At each surface inclination angle, α, a local coordinate system denoted by $(\tilde{O}, \tilde{x}, y, \tilde{z})$ is introduced for the purpose of obtaining the simplified solutions. The dynamic and microscopic contact angles are denoted as β and β_m, respectively. The letter a stands for the rivulet half width. By τ is denoted a point where the outer and inner solutions for $\tilde{h}(x, y)$ (rivulet profile with respect to local coordinate system) are stitched together. We also introduce the cylinder radius, R, and arc length coordinate, $s = \alpha R$.*

The compared simplified solutions to the system of the Navier-Stokes equations were obtained under the following assumptions,

1. The studied liquid is Newtonian, ρ, μ and γ are constant.
2. The rivulet profile shape is constant in time. Furthermore, rivulet liquid flow rate is assumed to be constant along the cylinder.
3. There is no shear between the gas and liquid phases.
4. The liquid velocities in the directions transversal and normal to the cylinder are negligible in comparison to the one in its longitudinal direction, $\tilde{u} \gg \tilde{v} \sim \tilde{w}$. The inertial effects can be neglected in y and $\tilde{z}$ directions.
5. The gravity is the only acting body force.
6. The gravity effects on the velocity of the contact lines are neglected. Also the dynamic contact angles, $\beta = \beta(s)$ are assumed small all along the rivulet.
7. There is a thin precursor film of height l on the whole studied surface. Thus there is no contact angle hysteresis and $\beta_m = 0$. The height of the precursor film, l, can also be taken as the intermediate region length scale well separating the inner and outer solution for the profile shape[5].

In the case of CFD, only the assumption of a shallow rivulet was retained and the solution was obtained using the lubrication approximation.

3. Simulation methods

As it was stated above, in overall four different methods were used to simulate a rivulet flowing down a slowly varying substrate. As a benchmark model, a CFD simulation carried out in OpenFOAM software was used. The simulation was based on solving the equation (1),

$$h_t - \frac{1}{3\mu}\nabla\left[\left(h^3 + 3\lambda h^2\right)(\rho g \nabla h - \gamma \nabla \kappa)\right] = 0, \qquad (1)$$

directly, using a finite volume method.

The remaining three methods were all obtained by solving a simplified version of the equation (1) analytically. The obtained profile shape function, $\tilde{h}$, for three cases of different substrate inclination angle, $\alpha < \pi/2$, $\alpha = \pi/2$ and $\alpha > \pi/2$ denoted as (i), (ii) and (iii) is,

$$\tilde{h}(\zeta) = \begin{cases} \dfrac{a\tan\beta}{\mathrm{B}}\left(\dfrac{\cosh \mathrm{B} - \cosh \mathrm{B}\zeta}{\sinh \mathrm{B}}\right) & (i) \\ \dfrac{a\tan\beta}{2}(1-\zeta^2) & (ii) \\ \dfrac{a\tan\beta}{\mathrm{B}}\left(\dfrac{\cos \mathrm{B}\zeta - \cos \mathrm{B}}{\sin \mathrm{B}}\right) & (iii) \end{cases}, \qquad (2)$$

where B is the Bond number of the problem, defined as $\mathrm{B} = a\sqrt{\rho g|\cos\alpha|/\gamma}$, representing the ratio of volume and surface forces in the rivulet and ζ is the y coordinate scaled by the rivulet half-width, $\zeta = y/a(s)$.

Using the stated assumptions, one can derive the relation for liquid volumetric flow rate in the form,

$$\frac{\mu Q}{a^4 \rho g \sin\alpha \tan^3\beta} = F(\mathrm{B}), \qquad (3)$$

where

$$F(\mathrm{B}) = \begin{cases} \dfrac{54\mathrm{B}\cosh\mathrm{B} + 6\mathrm{B}\cosh 3\mathrm{B} - 27\sinh\mathrm{B} - 11\sinh 3\mathrm{B}}{36\mathrm{B}^2\sinh^3\mathrm{B}} & (i) \\ \dfrac{4}{105} & (ii) \\ \dfrac{27\sin\mathrm{B} + 11\sin 3\mathrm{B} - 54\mathrm{B}\cos\mathrm{B} - 6\mathrm{B}\cos 3\mathrm{B}}{36\mathrm{B}^2\sin^3\mathrm{B}} & (iii) \end{cases} \qquad (4)$$

In the cases of a rivulet with fixed width or contact angle, the equations (2), (3) and (4) completely define the rivulet free surface shape.

In the case of a rivulet with varying both contact angle and width, those two quantities have to be bound together. Such a bounding based on the Cox-Voinov law is proposed, resulting in the differential equation for the rivulet half-width evolution,

$$\frac{\mathrm{d}a}{\mathrm{d}s} = \frac{2\gamma(R\sin\alpha - s\cos\alpha)\beta^3}{9\rho g l^2 R \sin^2\alpha \ln[a/(2e^2 l)]}. \qquad (5)$$

4. Results and Discussion

A comparison of the rivulet free surface shapes for the different methods is depicted in Fig. 2. Although all the results are qualitatively the same, the methods with either a or β fixed do not seem to be as close to the CFD solution as the method with both those properties kept variable.

Further argument in this direction can be based on the comparison of the Bond number evolution for the different methods. The agreement between method with variable β and a and CFD results is the strongest (consult right side of Fig. 3).

However, all the studied simplified methods predict higher rivulet profiles maximum than CFD. This may be a result of the necessity to impose some initial liquid velocity at $\alpha = 0$ in the case of CFD.

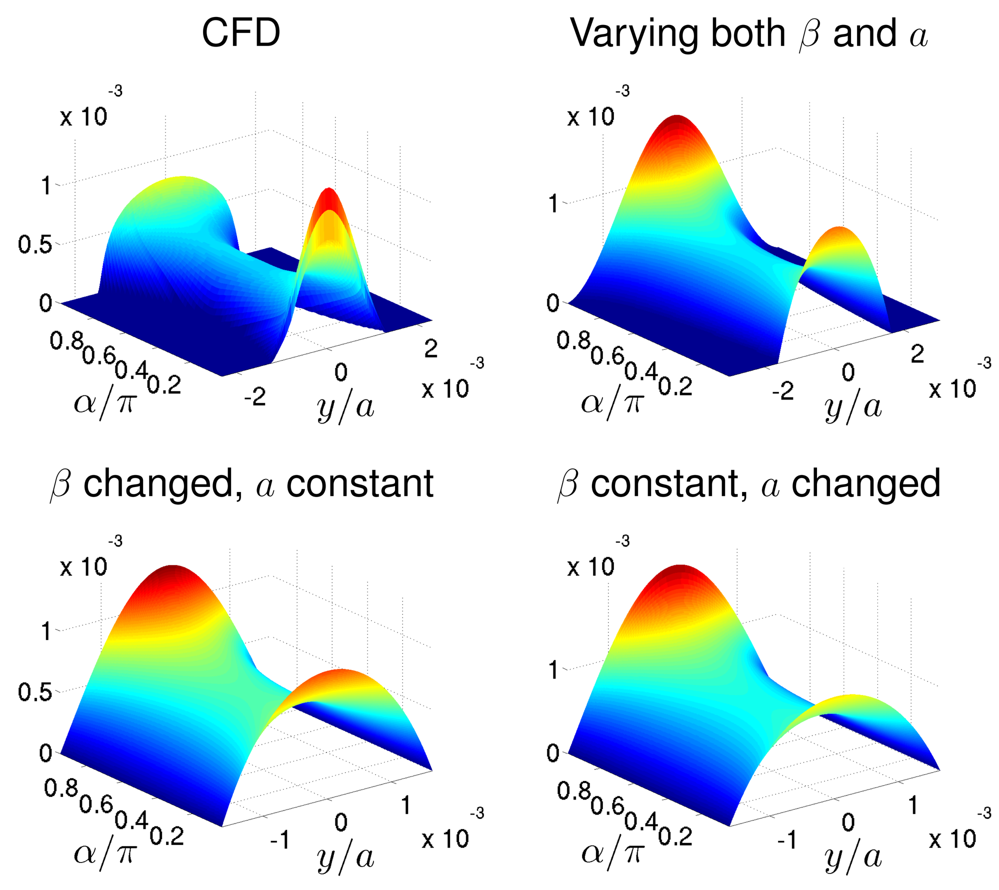

Figure 2: *Comparison of rivulet free surfaces projected on a plane for all the tested calculation methods. Case of the flow rate $Q = 1 \cdot 10^{-7}\,\mathrm{m^3s^{-1}}$ is depicted.*

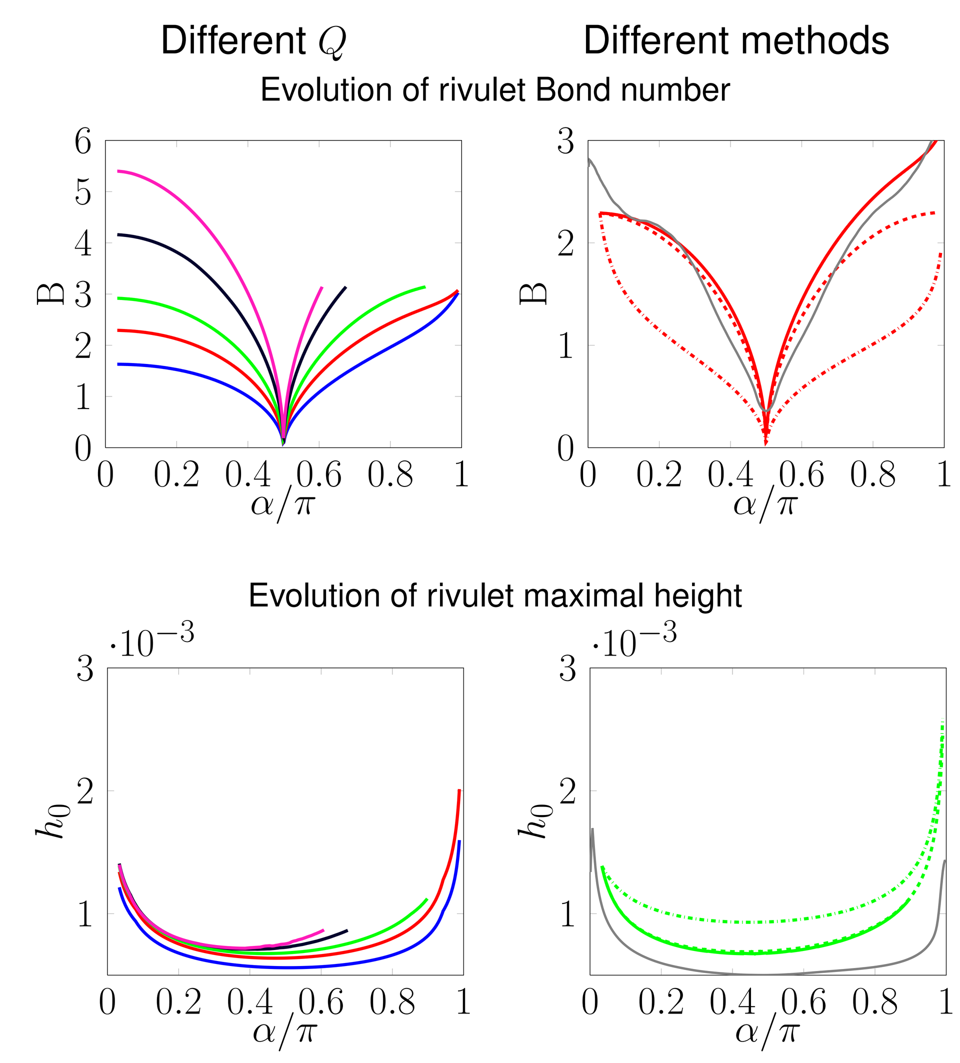

Figure 3: *Evolution of the rivulet Bond number and its maximal height along the cylinder. Five different flow rates, $Q = 1 \cdot 10^{-7}$ (—), $2 \cdot 10^{-7}$ (—), $3 \cdot 10^{-7}$ (—), $5 \cdot 10^{-7}$ (—) and $7 \cdot 10^{-7}$ (—) $\mathrm{m^3s^{-1}}$; and three calculation methods, were compared. Solid line (—) is used for simplified calculation with both β and a changed. Results for a and β kept constant are denoted by (····) and (- - -), respectively. Grey lines are used to depict the corresponding CFD results.*

In the left side of Fig. 3, an evolution of rivulet Bond number and maximal profile height is depicted for cases of a different flow rate. A line ending before $\alpha/\pi = 1$ indicates liquid dripping from the cylinder.

5. Conclusion

In the presented work, we provided a comparison between several rivulet modeling methods and CFD simulation. The simplified calculation with variation in both β and a seems to provide a rather good balance between the results accuracy and a method complexity.

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