Rektorys' competition 2014

Simplified model for a rivulet spreading down an inclined wetted plate

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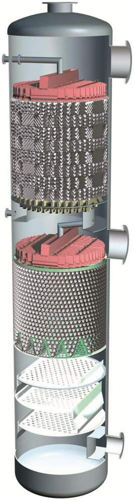
Outline

Why to study rivulets

Numerous applications in mass transfer and reaction engineering



[Sulzer ChemTech]

Hydrodynamics

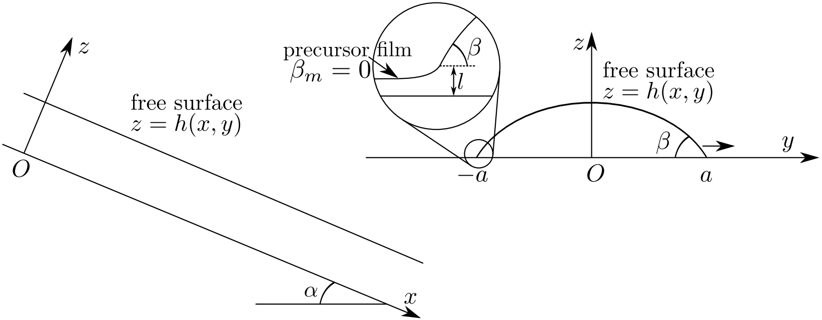
- Fuel cells
  - water management inside PEMFC fuel cells
- Aerospace engineering
  - in flight formation of rivulets on plane wings

Gas-liquid interface

- Packed columns
  - wetting performance
  - mass transfer coefficients
- Catalytic reactors
  - wetting of the catalyst

Used coordinate system

Cartesian coordinate system and basic notations



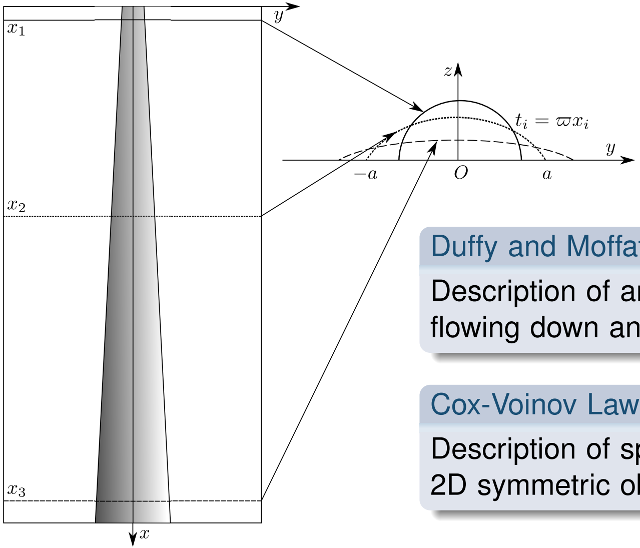
Notations

$a[\mathrm{m}]$ ......half-width of the rivulet
$h[\mathrm{m}]$ ....height
$l[\mathrm{m}]$ .. intermediate region length scale

$x, y, z[\mathrm{m}]$ ..... coordinate system
$\alpha[-]$ ...... plate inclination angle
$\beta[-]$ ......dynamic contact angle

Proposed method principle

Parallel between spreading of trickle in time and along the plate



Duffy and Moffat[11]

Description of an uniform rivulet flowing down an inclined plate

Cox-Voinov Law[24]

Description of spreading of the 2D symmetric object in time

Symplifying assumptions

Reduce problem to one spatial and one time coordinate

- Newtonian liquid, ρ, μ and γ are constant
- $h_t(t, x, y) = 0$, Q is constant
- $\mathbf{u} = (u, v, w)$, $u \gg v \sim w$
- $z = h : u_x = v_y = 0$
- Gravity is the only acting body force.
- The rivulet is shallow. Its dynamic contact angles are assumed small, $\beta(x) \ll 1$, and its GL interface is nearly flat, $h_y(x, y) \ll 1$.
- There is a thin precursor film of height l on the whole studied surface. Thus there is no contact angle hysteresis and $\beta_m = 0$. The height of the precursor film, l, can also be taken as the intermediate region length scale well separating the inner and outer solution for the profile shape[24].

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Outline

Simplified Navier-Stokes equations

Assumption of very shallow and nearly flat rivulet[11, 12]

Simplified NS

$$
\begin{aligned}
0 &= -p_x + \rho g \sin\alpha + \mu u_{zz} \\
0 &= -p_y \\
0 &= -p_z - \rho g \cos\alpha
\end{aligned}
$$

Used boundary conditions

$$
\begin{aligned}
z = \quad 0: &\quad u = u(y,z) = 0 \\
z = \quad h: &\quad p = p_A - \gamma h_{yy} \quad \text{and} \quad u_z = 0 \\
y = \pm a: &\quad h = 0 \quad \text{and} \quad h_y = \mp \tan\beta
\end{aligned}
$$

Notes

$$
h = h(y), \quad h_y = \frac{\mathrm{d}\, h}{\mathrm{d}\, y}, \quad p = p(y,z)
$$

Legend to results of integration

In the presented case, there exist a physical solution for $\alpha \in (0, \pi)$

Notation

The problem decomposes to three cases which will be denoted:

- $(i) \iff \alpha \in (0, \pi/2)$
- $(ii) \iff \alpha = \pi/2$
- $(iii) \iff \alpha \in (\pi/2, \pi)$

Dimensionless numbers

$$B = a^2 \frac{\rho g \cos|\alpha|}{\gamma}$$

$$\zeta = \frac{y}{a}$$

$\rho[\mathrm{kg\,m^{-3}}]$ ....liquid density

$\gamma[\mathrm{N\,m^{-1}}]$ ....surface tension

$g[\mathrm{m\,s^{-2}}]$ ....gravitational acceleration

Integration results

Rectilinear rivulet gas-liquid interface shape

Gas-liquid interface shape for the three cases

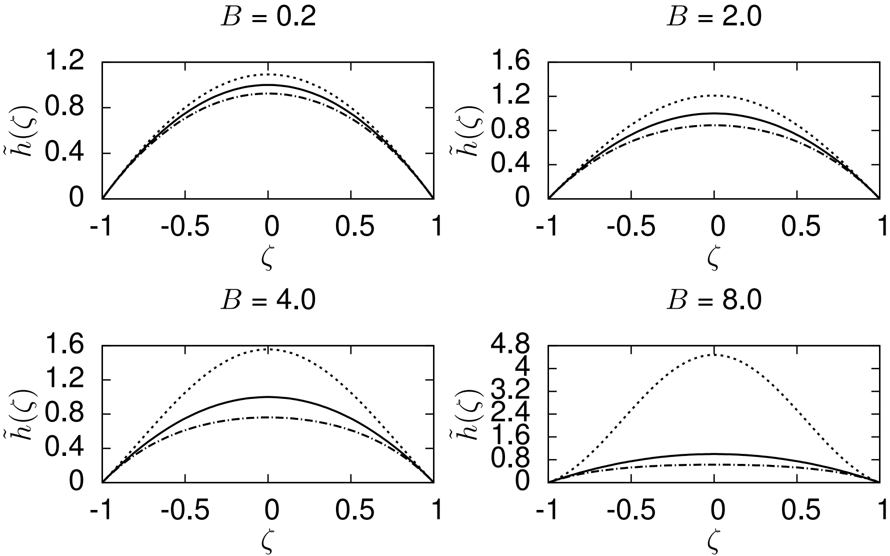
$$h(\zeta) = \begin{cases} \dfrac{a\tan\beta}{\sqrt{\mathrm{B}}}\left(\dfrac{\cosh\sqrt{\mathrm{B}} - \cosh\sqrt{\mathrm{B}}\zeta}{\sinh\sqrt{\mathrm{B}}}\right) & (i) \\ \dfrac{a\tan\beta}{2}(1-\zeta^2) & (ii) \\ \dfrac{a\tan\beta}{\sqrt{\mathrm{B}}}\left(\dfrac{\cos\sqrt{\mathrm{B}}\zeta - \cos\sqrt{\mathrm{B}}}{\sin\sqrt{\mathrm{B}}}\right) & (iii) \end{cases}$$

Scaling of the GL interface height

$$\tilde{h}(\zeta) = \frac{2h(\zeta)}{a\tan\beta}$$

Effects of Bond number on GL interface shape

Dependence of GL interface shape on the ratio of surface and volumetric forces



Liquid volumetric flow rate

Integration of velocity field on a domain of one transversal cut

Velocity field integration

$$\frac{Q}{a} = \int_{-1}^{1} \int_{0}^{h(\zeta)} u(\zeta, z)\,\mathrm{d}z\,\mathrm{d}\zeta = \int_{-1}^{1} \int_{0}^{h(\zeta)} \frac{\rho g \sin\alpha}{2\mu} \left(2h(\zeta)z - z^2\right)\,\mathrm{d}z\,\mathrm{d}\zeta$$

Dimensionless liquid flow rate

$$\frac{\mu Q}{a^4 \rho g \sin\alpha \tan^3\beta} = F(\mathrm{B})$$

$$F(\mathrm{B}) = \begin{cases} \dfrac{54\sqrt{\mathrm{B}}\cosh\sqrt{\mathrm{B}} + 6\sqrt{\mathrm{B}}\cosh 3\sqrt{\mathrm{B}} - 27\sinh\sqrt{\mathrm{B}} - 11\sinh 3\sqrt{\mathrm{B}}}{36\mathrm{B}^2\sinh^3\sqrt{\mathrm{B}}} & (i) \\ \dfrac{4}{105} & (ii) \\ \dfrac{27\sin\sqrt{\mathrm{B}} + 11\sin 3\sqrt{\mathrm{B}} - 54\sqrt{\mathrm{B}}\cos\sqrt{\mathrm{B}} - 6\sqrt{\mathrm{B}}\cos 3\sqrt{\mathrm{B}}}{36\mathrm{B}^2\sin^3\sqrt{\mathrm{B}}} & (iii) \end{cases}$$

Liquid flow rate scaling and properties

Maximal flow rate occurs at $\alpha = \pi/2$

Scaling

$$\tilde{Q} = \frac{105\mu}{4a^4\rho g \sin\alpha \tan^3\beta} Q = \frac{105\rho g\mu \cos^2\alpha}{4\gamma^2 \sin\alpha \tan^3\beta} \frac{Q}{\mathrm{B}^2}$$

Asymptotic behavior

$$(i): \quad \lim_{\mathrm{B}\to\infty} \tilde{Q}(\mathrm{B}) = 0$$

$$(iii): \quad \lim_{\mathrm{B}\to\pi\frac{2}{-}} \tilde{Q}(\mathrm{B}) = \infty$$

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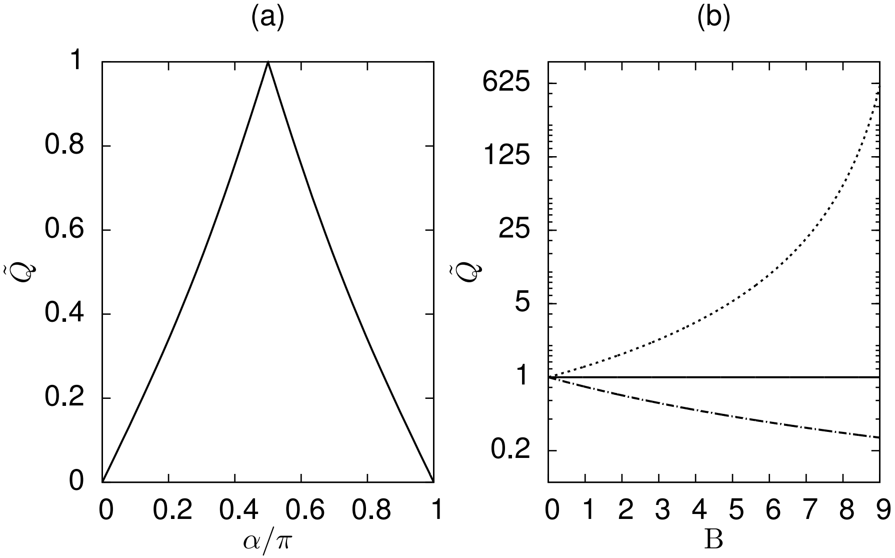


Outline

Difference from static rivulet

Changed boundary conditions

Simplified NS

$$
\begin{aligned}
0 &= -p_x + \rho g \sin\alpha + \mu u_{zz} \\
0 &= -p_y \\
0 &= -p_z - \rho g \cos\alpha
\end{aligned}
$$

Used boundary conditions

$$
\begin{array}{rlcr}
z = \quad 0: & \mathbf{u}(x,y,z) + \lambda\nabla\mathbf{u} = \mathbf{0}, & & \lambda \ll 1 \\
z = \quad h: & p = p_A - \gamma\kappa & \text{and} & u_z = 0 \\
y = \pm a(x): & h = 0 & \text{and} & h_y = \mp \tan\beta(x)
\end{array}
$$

Notes

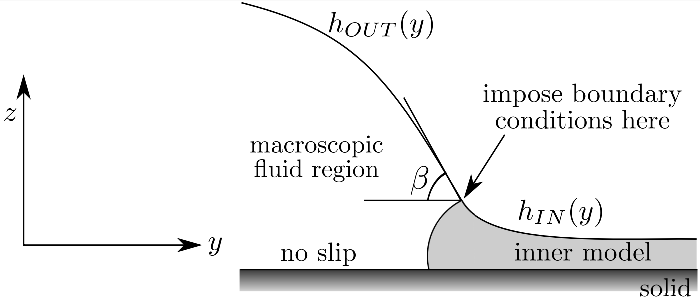
$$
h = h(x,y), \quad h_y = \frac{\partial h}{\partial y}, \quad p = p(x,y,z)
$$

Cox-Voinov law

Solution of thin film governing equation for spreading of symmetric object[24]

Thin film governing equation - outer and inner

$$h_t + \frac{\gamma}{3\mu}\frac{\partial}{\partial a}\left(h^3 h_{aaa}\right) = 0, \qquad h_t + \frac{1}{3\mu}\frac{\partial}{\partial a}\left[\left(h^3 + 3\lambda h^2\right)\gamma h_{aaa}\right] = 0$$

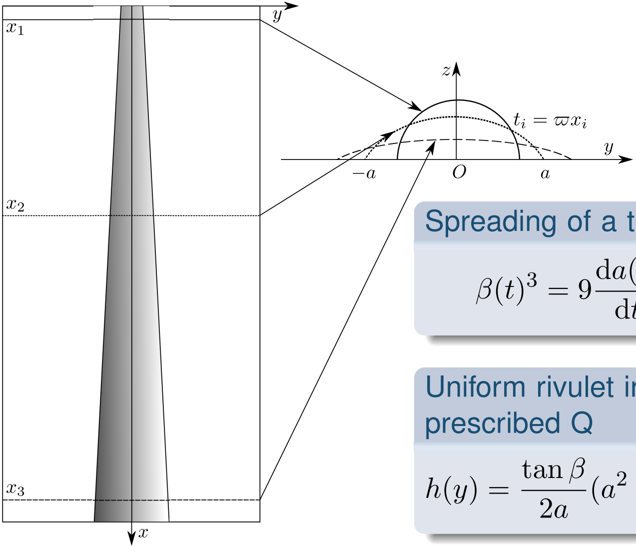


Cox-Voinov law[24]

$$\beta(t)^3 = 9\frac{\mathrm{d}a(t)}{\mathrm{d}t}\frac{\mu}{\gamma}\ln\left(\frac{a(t)}{2e^2 l}\right)$$

Basic principle remainder

Combination of Cox-Voinov law with unidirectional rivulet flow description



Spreading of a trickle in time

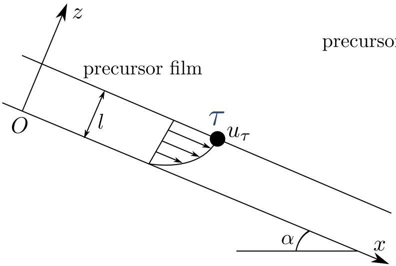
$$\beta(t)^3 = 9\frac{\mathrm{d}a(t)}{\mathrm{d}t}\frac{\mu}{\gamma}\ln\left(\frac{a(t)}{2e^2 l}\right)$$

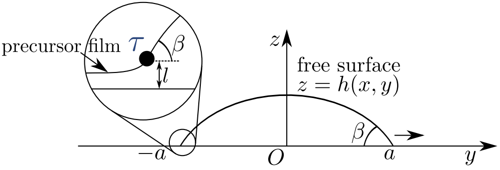
Uniform rivulet interface with prescribed Q

$$h(y) = \frac{\tan\beta}{2a}(a^2 - y^2), \quad a \approx \eta\frac{1}{\beta^{3/4}}$$

Transformation from t to x

Presence of falling thin liquid film, l, on all the plate





From t to x using u_τ

$$u_\tau = \frac{\rho g \sin\alpha}{2\mu} l^2 \implies t = \frac{2\mu}{\rho g l^2 \sin\alpha} x = \varpi x$$

Modeling method for the case of no gravity effects

Gas-liquid interface shape defined implicitely by 1 NAE

- $\beta \ll 1 \implies a \doteq \eta \frac{1}{\beta^{3/4}}$
- $$\beta(t)^3 = 9\frac{\mathrm{d}a(t)}{\mathrm{d}t}\frac{\mu}{\gamma}\ln\left(\frac{a(t)}{2e^2 l}\right) \rightsquigarrow \beta^{19/4} = -A\frac{\mathrm{d}\beta}{\mathrm{d}t}\ln\left(\frac{B}{\beta^{3/4}}\right)$$
$$\beta = \beta(t), \quad A = \frac{27}{4}\frac{\eta\mu}{\gamma}, \quad B = \frac{\eta}{2e^2 l}, \quad \eta = \left(\frac{4\mu Q}{105\rho g \sin\alpha}\right)^{\frac{1}{4}}$$
- $$\beta(0) = \beta_0, \quad t = \varpi x \rightsquigarrow x - \frac{\bar{A}}{\beta^{15/4}}\left[\ln\left(\frac{B}{\beta^{3/4}}\right) - \frac{1}{5}\right] + \bar{C} = 0$$
$$\bar{A} = \frac{4}{15}\frac{A}{\varpi}, \quad \bar{C} = \frac{4A}{15\varpi\beta_0^{15/4}}\left[\ln\left(\frac{B}{\beta_0^{3/4}}\right) - \frac{1}{5}\right]$$

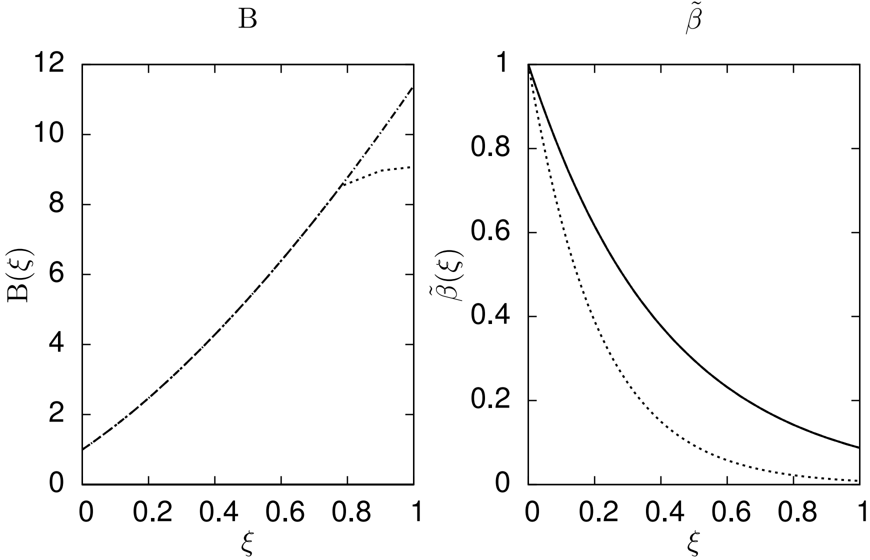
Modeling method gravity effects

1 ODE with 1 NAE nested in each time step has to be solved numerically

- $t = \varpi x, \quad \beta(0) = \beta_0$
- $$\frac{\mu Q}{a^4 \rho g \sin\alpha \tan^3\beta} = F(\mathrm{B})$$
- $$\beta(\varpi x)^3 = \frac{9}{\varpi} \frac{\mathrm{d}a(\varpi x)}{\mathrm{d}x} \frac{\mu}{\gamma} \ln\left(\frac{a(\varpi x)}{2e^2 l}\right)$$

B and β along the plate

$\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1\,\mathrm{m}$, $\beta_0 = 0.05$, $Q = 0.01\,\mathrm{ml\,s^{-1}}$, $\alpha/\pi = \{1/3, 2\pi/3\}$



Outline

Velocity in x-axis direction

Approximated through the static rivulet description

Dimensional $u(\mathbf{x})$

$$\mathbf{u}(\mathbf{x}) = (u(\mathbf{x}), v(\mathbf{x}), w(\mathbf{x}))$$

$$u(\mathbf{x}) = \frac{\rho g \sin\alpha}{2\mu}\left(2h(x,y)z - z^2\right)$$

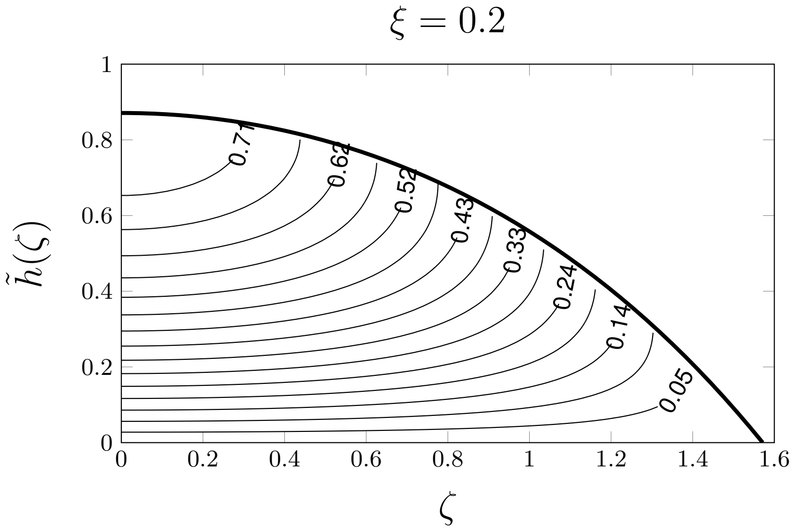
Scaling

$$\xi = \frac{x}{L}, \quad \zeta = \frac{y}{a(0)}, \quad \tilde{z} = \frac{z}{h(0,0)}$$

$$\tilde{u} = \frac{u}{u(\mathbf{0})}$$

Contours plot of velocity in x-axis direction

Water, $\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1\,\mathrm{m}$, $\beta_0 = 0.05$, $Q = 0.01\,\mathrm{ml\,s^{-1}}$, $\alpha = \pi/3$



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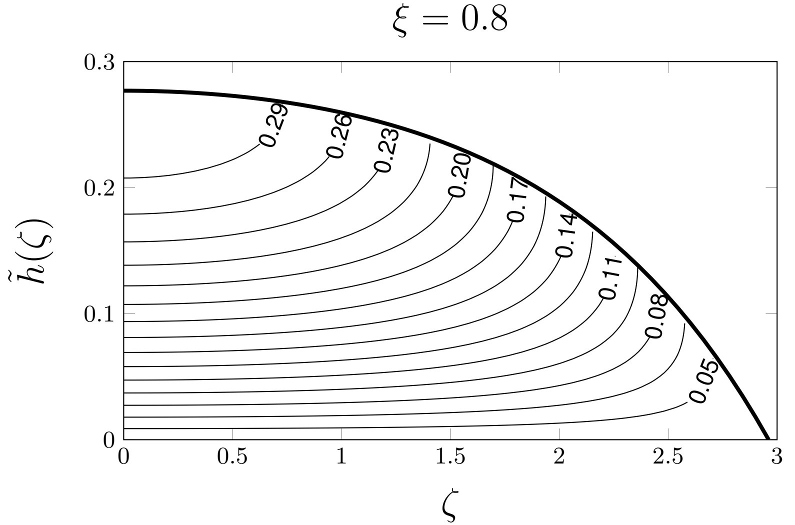


Simulation of velocity field in transversal cut

Numerical PIV based technique – connect the dots

For each subrivulet:

- Get the current width of the rivulet, ζ_i, from the Cox-Voinov law.
- Calculate the current maximal rivulet height, $\tilde{h}_i^0$, from description of an uniform rivulet.
- Create a mesh Ω_i^h on a domain Ω_i,

$$\Omega_i = \left\{ (\zeta, \tilde{z}) \ : \ \zeta \in \langle 0; \zeta_i \rangle \mid \tilde{z} \leq \tilde{h}_i(\zeta) \right\}$$

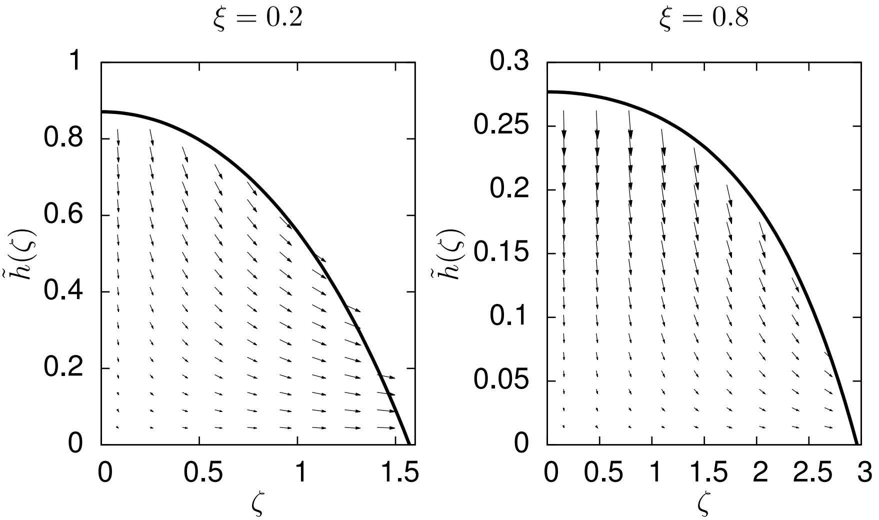
$$\Omega_i^h = \left\{ (\zeta^j, \tilde{z}^k) \ : \ \zeta^j \in \langle 0; \zeta_i \rangle \ \middle| \ \tilde{z}^k \leq \tilde{h}_i(\zeta^j) \right\}_{j=1,\dots,M_1;\, k=1,\dots,M_2}$$

- Save the current mesh, Ω_i^h.

From the saved meshes, Ω_i^h, $i = 1, \dots, N$, evaluate the velocity field in the $\zeta - \tilde{h}$ plane (connect the dots).

Simulated velocity field v and w components

Water, $\xi = x/L$, $\tilde{\beta} = \beta/\beta_0$, $L = 0.1\,\mathrm{m}$, $\beta_0 = 0.05$, $Q = 0.01\,\mathrm{ml\,s^{-1}}$, $\alpha = \pi/3$



Conclusions and Outlook

Description of a spreading rivulet with no need of PDE solving

Practical advantages of proposed algorithm

- When the gravity effects can be neglected, the method is suitable for parallelization
- Usable for qualitative study of dependence of S_{g-l} on process parameters

Accuracy

- Deviation $< 5\,\%$ in comparison with experimental data[27].

Outlook

- Study of spreading over an edge of the plate (discontinuous change of α)

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In order of appearance

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